“YOU’RE GOING TO WANT TO FIND OUT WHICH AND PROVE IT”: COLLECTIVE ARGUMENTATION IN A MATHEMATICS CLASSROOM

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Abstract

One of the proposals of the North American educational reform movement is that teachers should stress scientific argumentation more than the manipulation of symbols and algorithms in their mathematics instruction. The aim of this article is to apply some theoretical concepts, drawn from the fields of sociolinguistics and rhetoric, to the analysis of argumentation in a lesson conducted in an urban middle school classroom. Our analysis focuses on the implementation of the classroom teacher’s instructional goals during a lesson on area measurement. As a result of our analyses, we found that she achieved her instructional goals of being nondirective in her teaching and getting students actively involved in arguing about mathematical concepts. The teacher was able to orchestrate discussion by recruiting attention and participation from her class, aligning students with argumentative positions through reported speech, highlighting positions through repetition, and pointing out important aspects of their arguments through expansion. In addition, we also found that her students differed in the way they framed the mathematical content of the lesson in terms of the facts or grounds, algorithms or warrants, premises or backings, as well as solutions or claims. Their arguments also varied in terms of explicitness and ability to integrate their classmates’ arguments. In conclusion, we feel that discourse analysis, based on sociolinguistic and rhetorical theoretical frameworks, can be a

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valuable tool for the evaluation of educational reform in mathematics. © 1998 Elsevier Science Ltd. All rights reserved.

Introduction

Ms. Kingsley*, a middle school mathematics teacher in an urban public school in the United States and a participant in an educational reform project, is beginning a lesson on area measurement by asking for volunteers to go to the overhead projector and explain their answers to one of the problems assigned. She indicates that she has surveyed the classroom and has found that its members disagree about the problem’s solution. In her request, she adds the condition that, “you’re going to want to find out which (answer is correct) and prove it.” This approach to beginning a mathematics lesson differs from more familiar classroom scenes in which middle school students are expected to use algorithms to solve familiar problems but not to prove the validity of their solutions through discussion with their classmates. Yet it is consistent, we argue, with the proposal of some educational reformers that instruction in mathematics should resemble the practice of mathematicians. In this way, reformers are asking that we consider classrooms to be communities that could resemble, in some ways, the communities of practicing mathematicians.

Lampert (1990); Cobb & Bauersfeld (1995), for example, maintain that learning how to support one’s mathematical conclusions through the use of logical argumentation should augment or even replace the (often meaningless) manipulation of symbols and algorithms in classrooms. Lampert articulates what she means by argumentation in mathematics when she describes the “zig-zag path” (p. 30) by which mathematical conclusions are derived. This path begins with conjecture, involves the examination of premises, and includes disagreement and counter-arguments. Cobb and Bauersfeld use the distinction between inquiry and school mathematics to represent the differences between classrooms that foster genuine discussion of mathematical ideas and those that encourage rote symbol manipulation.

This viewpoint is also reflected in the new standards for mathematics instruction from the National Council of Teachers of Mathematics (1989, 1991) (NCTM). The NCTM standards propose, among other things, new expectations for what students should learn and about how this learning should occur. They mandate increased emphasis upon complex problem solving, higher level reasoning, making connections across mathematical domains, and communication. In addition, they recommend decreased emphasis upon rote computation, routine problem solving, and over-dependence upon the teacher or text for explaining and evaluating concepts and procedures. In other words, instruction should de-emphasize the transmission of factual or otherwise incontestable information from teachers to students and should encourage the active involvement of students in discussing ideas, making convincing arguments, reflecting on and clarifying their thinking. Educational reforms similar to those being tried in the United States are also occurring in Europe. For example, one approach being used in The Netherlands, Realistic Mathematics (Streefland,

*The names of the teacher and her students are pseudonyms.
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(1991), stresses an interactive learning process by which students construct their interpretations of problems, often in collaboration with others, and discuss multiple approaches to problem solutions.

The new developments in education are strikingly consistent with current trends in the philosophy and sociology of science. One of the major goals of recent analyses of scientific practice (Latour & Woolgar, 1986; Pera, 1994; Krips et al., 1995) is to highlight the central place of argumentation in the process of achieving agreement about the nature of scientific objects. For example, in their analysis of the laboratory of the Salk Institute, Latour & Woolgar (1986) demonstrate that scientific objects emerge through a process of examination in the scientific community that involves initial speculation, subsequent refutations of findings based on the questioning of assumptions and methods, increasing or decreasing certainty based on replication or failure to do so, and finally the widespread acceptance of findings as no longer subject to discussion. This conception of science, which depends as much on the ability to persuade members of a scientific community as it does upon the use of the scientific method, departs significantly from prior views of scientific discovery as a purely individual enterprise. Thus, educational reformers and philosophers of science are proposing that teaching and learning be viewed as occurring in a social context.

We believe that this new view of science provides educational researchers with a rationale for examining the use of argumentation in classrooms. While research about scientific practice can justify an interest in studying argumentation, research in the fields of sociolinguistics and rhetoric can provide the methodological tools. Some of the analytic procedures that sociolinguists and rhetoricians have been applying to the study of everyday conversational practices are now being employed by educational researchers studying argumentation in inquiry mathematics classrooms (e.g. O’Connor & Michaels, 1993, 1996; Krummheuer, 1995). The analytic tools that will be used in this article include notions of frame, contextualization cues, and discussion orchestration from sociolinguistics and claims, grounds, warrants, and backings from rhetoric.

The aim of this article is to use these analytic tools to explicate a transcript of discourse that was recorded in Ms. Kingsley’s classroom on the day she asked her students to supply proof for their solutions to the area problem. The goal of our analysis of Ms. Kingsley’s classroom will be an understanding of the socialization of argumentation in her mathematics classroom. A secondary aim of this work is to provide teachers and teacher educators with a detailed picture of argumentation in this classroom. If we expect teachers to change their practice in the light of the proposals of the educational reformers, then we will need examples of the functioning of successful inquiry mathematics classrooms. We hope that the analysis presented here will contribute to the corpus of examples being generated by inquiry mathematics projects in the United States and Europe (cf. Lampert, 1990; Streefland, 1991; O’Connor & Michaels, 1993, 1996; Cobb & Bauersfeld, 1995; Forman, 1996).

Sociolinguistics Research and the Socialization of Argumentation

Unlike practicing mathematicians, students must also learn about the nature of mathematical argumentation as they attempt to use it to gain an understanding of the mathemat-
ical objects under discussion. While students have previous experience with arguments in their everyday life, each professional field imposes its own set of norms for argumentation (Toulmin, 1958). Argumentation in classrooms depends upon the members of the classroom community coming to share a common perspective on the objects discussed (both actual and symbolic) and on the means by which the discussion can occur (Van Oers, 1996). This shared perspective, in turn, depends upon the expectations of the community members. One could predict that many middle school students come to inquiry mathematics classrooms with expectations about arguments that would clash with the norms of mathematical argumentation. That is, they may believe that argumentation is unnecessary because there is only one correct solution strategy to every problem (which the teacher can or should provide). In addition, they may have presuppositions about the norms of school mathematics instruction that would contradict those of inquiry mathematics. For example, they may expect that speed and accuracy would be more important than conceptual understanding. Thus, if teachers want to help students engage in argumentation similar to that used by mathematicians, then they will have to intervene to change students’ expectations about and knowledge of argumentation in each specific problem context.

One way to understand the development of shared expectations among community members is to use the sociolinguistic concept of frame. Tannen & Wallat (1993) propose that frame can be used as a noun to refer to a person’s knowledge structure or as a verb to refer to a way of interpreting messages. “Frames are the organizational and interactional principles by which situations are defined and sustained as experience” (Goffman, 1974, cited in Schiffrin, 1994, p. 104). One of the earliest applications of frame was employed by Bateson (1972) when he argued that movements between playful and serious activities involve a change in the way an activity is framed by the participants. Tannen cautions us, however, not to assume that frames are static, unchanging interpretive procedures or knowledge structures—frames are constantly being revised during conversation.

Schiffrin proposes that people communicate their expectations or frames through what Gumperz called “contextualization cues” (Schiffrin, 1994, p.99), which serve as conversational metamessages (cf. Tannen, 1993). “Contextualization cues (are) aspects of language and behavior (verbal and nonverbal signs) that relate what is said to the contextual knowledge… that contributes to the presuppositions necessary to the accurate inferencing of what is meant” (Schiffrin, 1994, pp. 99–100). She argues that contextualization cues serve as frequent, although often unconscious, signals of the speaker’s interpretation of the propositional content of the message. When speakers and listeners interpret a situation similarly, then their interaction appears effortless (i.e. few misunderstandings occur). Thus, misunderstandings or overt indices of conflict can indicate that participants are not framing the situation in the same way.

Two papers by O’Connor & Michaels (1993, 1996) apply these sociolinguistic notions to the study of inquiry mathematics classrooms. They focus on the means by which teachers socialize argumentation through their orchestration of classroom discussions. Because our analysis is based, in part, upon this work by O’Connor and Michaels, we will discuss their perspective in some detail. O’Connor and Michaels highlight the importance of students’ active participation in the classroom community by assuming, with the teacher’s assistance, various roles in ongoing debates about academic material. They argue, along the lines of the NCTM standards, that higher-level thinking is fostered by students’ involvement in taking and defending a particular position on a controversial topic against
the claims of other students. This instructional process depends upon the skillful orchestration of classroom discussion by the teacher. Discussion orchestration “provides a site for aligning students with each other and with the content of the academic work while simultaneously socializing them into particular ways of speaking and thinking” (O’Connor & Michaels, 1996, p. 65).

How does the teacher (as well as the students) accomplish this alignment of people and positions, thereby creating a shared frame for interpreting an argument? O’Connor and Michaels focus on the notion of revoicing to illustrate this process. Revoicing involves the reuttering of another person’s speech through repetition, expansion, rephrasing, and reporting. They propose that revoicing serves to reformulate and create alignments and oppositions in an argument. Revoicing is a broad category and includes the following goals: to clarify or amplify content; to explain reasoning further; to introduce particular ideas; or to redirect the discussion. Creating alignment and oppositions means that the teacher “is positioning the student in relation to the content, conferring on (or attributing to) that student a stance with respect to the topic under discussion, a stance the student may be only dimly aware of” (O’Connor & Michaels, 1996, p. 76). In this way, teachers not only put claims in students’ mouths, they also assign a particular role to students in the overall debate. In so doing, they create the debate that is not only comprised of conceptual positions but also worthy (or unworthy) protagonists in an argument. In essence, the teacher can reframe the classroom discussion both in terms of content and in terms of social roles. Revoicing by the teacher may change the way students “see themselves and each other as legitimate participants in the activity of making, analysing, and evaluating claims, hypotheses, and predictions” (O’Connor & Michaels, 1996, p. 78). Thus, revoicing functions as a contextualization cue: a metamessage that signals something about how the argumentation activity is framed by the teacher.

As we have seen, notions from sociolinguistics such as frame, contextualization cues, and discussion orchestration provide a framework for understanding how a shared definition of an argument can be obtained through subtle signals such as revoicing. What is missing from the sociolinguistic analysis of discussion orchestration is the propositional content of the argument. That is, when a teacher expands a student’s utterance, for example, we need to know what is important about this expansion in terms of the structure of the mathematical argument being presented. For this additional analysis we have turned to some rhetorical concepts.

Rhetorical Analysis and the Socialization of Argumentation

Building upon previous work by Krummheuer (1995), we define arguments as “the intentional explication of the reasoning of a solution during its development or after it” (Krummheuer, 1995, p. 231), and adopt Toulmin’s method of argument analysis to identify the components of the students’ arguments. According to Toulmin (1958); Toulmin et al. (1979), the utterances that make up an argument have different functions which can be classified into four categories: claims, grounds, warrants, and backings.
Claims

Claims are the assertions in need of argumentative support either because they have been explicitly or implicitly called into question by the audience or because the rules that govern the social interaction require participants to provide support for the veridicality of their accounts. Claims can be questioned, for example, in the context of a discussion when two or more participants disagree with each other. In inquiry mathematics classrooms, students may provide different solutions to a problem which would constitute competing claims.

Grounds

Grounds, on the other hand, are “the facts we appeal to as a foundation for the claim” (Toulmin, 1958, p. 97). Grounds need not be facts in the empirical sense, but statements that we take as givens under the assumption that they will not be challenged. Grounds that are challenged then become claims in a side argument until their status as facts is accepted or rejected by the classroom community.

Warrants

Warrants are utterances that explicate why certain grounds are relevant for the claim in contention and which entitle us to draw conclusions from accepted givens. In discussions in mathematics classrooms, warrants may be formulas or algorithms that allow us to find the values of unknown variables from the values of known variables (Toulmin et al., 1979).

Backings

Finally, the backings of an argument are utterances that strengthen the acceptability of warrants so that the connection between grounds and claims is not further scrutinized. In a sense, backings constitute the frame within which warrants hold as valid inferences (Toulmin, 1958; Krummheuer, 1995).

For example, a student may claim that the area of a particular rectangle is 20 cm². If her claim is challenged, then she may refer to the dimensions of the rectangle (5 cm by 4 cm) as her grounds. If the grounds are not challenged, then she may provide as her warrant the algorithm for computing area as length multiplied by width (or alternatively, she may partition the rectangle into two smaller rectangles say 4 by 4 cm and 1 by 4 cm and add their individual areas). Her backing for those warrants would presumably be that length times width is the correct algorithm for area (or that the sums of the areas of the enclosed rectangles must total to the area of the enclosing rectangle). Challenges to her claim would have to focus on the warrant and/or backing provided in support of it (provided that the grounds are not under dispute).

An important aspect of argumentation is that the selection of specific grounds, warrants, and backings is dependent upon the social context in which an argument is provided (Krummheuer, 1995). For instance, successful grounds for a particular claim are determined by their common acceptance without further questioning by the various participants in a debate. Likewise, successful backings are usually points of agreement among the
discussants in the sense that they constitute theories, beliefs, or more generally perspectives shared by those who are involved in the debate. Thus, the success of an argument depends not only on the correct application of an algorithm or logical soundness, but also on the degree to which a given set of grounds, warrants, and backings can actually convince the audience about the veridicality of a claim.

We argue that the teacher’s role in orchestrating through revoicing and other means is crucial since he or she is the ultimate judge in the classroom context of the criteria necessary to establish the veridicality of a claim. In other words, teachers’ framing of the criteria for adequate or appropriate argumentation influences the persuasiveness of students’ arguments. Teachers indicate their implicit as well as explicit criteria for arguments through contextualization cues such as revoicing. Thus, we propose that two analytic frameworks are useful for understanding the nature of arguments in Ms. Kingsley’s classroom. One framework from sociolinguistics allows us to examine how she orchestrates classroom discussions. The second from rhetoric permits us to focus on the propositional content of students’ and teachers’ contributions to the collective argument.

An Illustration of Collective Argumentation in Ms. Kingsley’s Classroom

In our discussion of the notion of frame, we indicated that frames are aspects of people’s expectations about social contexts. Frames emerge from past experiences with familiar contexts and are modified based on interpretations of social events as they unfold. In order to predict how a classroom lesson may be framed by participants, one needs to know something about the communicative intentions of some if not all the participants and the past history of similar lessons in this classroom. Information about both of these aspects of the lesson’s context are presented next.

The Teacher’s Instructional Goals

Ms. Kingsley had been participating in the QUASAR project for nearly two years at the time of the target lesson (May, 1992). QUASAR is a national reform project aimed at fostering and studying the implementation of improved mathematics programs in middle schools (typically grades 6–8 with students 13–15 years of age) that serve economically disadvantaged communities (Silver & Stein, 1996). The target lesson occurred in a sixth grade mathematics classroom that is regularly taught by Ms. Kingsley. The data that serve as the basis for the present investigation were collected as part of the project’s documentation of instructional practice (Stein et al., 1991). As a member of an educational reform project, Ms. Kingsley was familiar with the NCTM new standards and with the need to de-emphasize rote symbol manipulation and to emphasize active student construction of mathematical concepts as well as to foster mathematical communication.

She was interviewed before the lesson that is the focus of our analyses about her goals and plans. Her statements about her intended goals help us make some predictions about the shared expectations or frames that may emerge in the analysis of the discourse in her classroom. Our analysis may also reveal whether those goals were being implemented in
the segment examined. All direct quotations from Ms. Kingsley were obtained from QUASAR documentation interview transcripts.

Before the target lesson, Ms. Kingsley expressed her intention to involve the students in discussing mathematical concepts by using open-ended questions: “I hope I’ll do good questioning. I hope that I won’t be directive. I’ll try to have the kids go through it instead of telling them. You know, give them opportunities to construct it.” She also said she hoped to foster mathematical reasoning by encouraging her students to explore the generality of their knowledge and by calling attention to student generalizations:

I’m trying to work on pulling generalizations a little bit better… I think that I’m not doing a good enough job of getting the kids to sum some things up. It doesn’t mean I state it, but without really getting some kids to state some kinds of generalization that they’re coming to and really calling attention to what they’re saying… because sometimes they’ll say it and I don’t call enough attention to it. And so the other kids really miss that somebody’s come to a conclusion and maybe it’s something that does work.

Thus, Ms. Kingsley expressed her intentions to foster but not direct classroom discussion by having students summarize and generalize their thinking. She also hoped to direct attention to student explanations in order to help students reflect on their classmates’ thinking. If her goals were successfully implemented, then one would expect to see evidence of shared instructional responsibilities between this teacher and her students, and evidence of students discussing the generality of mathematical concepts rather than solely focusing on the accurate use of algorithms.

The Classroom History of the Area Lesson

Ms. Kingsley’s class had studied linear measurement before they began the lesson on area measurement that was the subject of our analyses. In some of those lessons, students had been asked to convert linear units measured in centimeters to units measured in millimeters and to identify the ratio that was needed in the conversion algorithm. In those lessons, the conversion ratio was found to be 1:10. On the day prior to the target lesson, students were asked to convert area units measured in centimeters squared to units measured in millimeters squared. Some students that day based their conversions on a 1:10 ratio while others based theirs on a ratio of 1:100. Thus, their prior history with conversion ratios may have led some students to over-generalize the 1:10 ratio from linear measurement to area measurement. Ms. Kingsley’s focus on generalization in her instructional goals may indicate her hope that students would question whether the 1:10 ratio could be applied to area measurement.

Immediately prior to the beginning of the lesson analysed below, students met in pairs or small groups to discuss their answers to a series of homework problems on area measurement. One of those problems was selected by the teacher for discussion in the segment that was the focus of our analysis. The problem asked the students to find the area of an irregular geometric figure by counting the number of whole and partial grid squares (each representing 1 cm²) that were enclosed by it, and to convert the result from centimeters squared (17 1/2 cm²) to millimeters squared. Thus, the focus of the lesson was not on the computation of the area of this figure (about which everyone in the class agreed) but, like the previous day’s lesson, the target lesson emphasized the understanding of the conversion of one area unit to another. If the conversion ratio used was 1:100, then the
solution would be 1750 mm²; if the ratio used was 1:10 (as in linear measurement), then the solution would be 175 mm². During the lesson, students, one at a time, presented their ideas about the relationship between different measures of area at the overhead projector placed in front of the class. The teacher stood at the back of the room and orchestrated the lesson from there. While arguments that occur during whole-class discussions are collective productions, we will attribute particular positions to student speakers at the overhead projector or in their seats who articulated elaborated explanations.

Invitation and Initiation: Larry’s Argument

At the beginning of the activity, the teacher opens the discussion by asking for a volunteer to go to the overhead projector, and give and justify his or her answers, in millimeters squared, to the area question*. 

10 Teacher: Okay is there somebody who’d go up, um, with what they did on the first one for the millimeter squared because I see a lot of different answers on this and you’re going to want to find out which and prove it. Okay.

By this invitation, the teacher not only legitimizes the existence of diverging positions with regard to the task at hand (“different answers”), but also sets the rhetorical and social norms that the participants will be expected to observe throughout the debate; namely, they will be required both to express their solution to the area problem and explain it convincingly (“prove it”) to the rest of the class. Thus, this opening query creates not only a space for discussion and difference, but also, and most importantly, a space where differences need to be resolved through argumentation. Finally, her opening query also defines the motivational conditions for the argument: that each participant will “want to” find out the most adequate solution by providing a convincing explanation. Larry is the first student to offer a solution.

11 Larry: 17½ centimeters.

12 Teacher: 17½ squared centimeters, you mean?

13 Larry: Yeah.

14 Teacher: Okay.

15 Larry: And a millimeter is one squared centimeters. I got 17 times 10.

16 Teacher: So you multiplied by 10. Why did you multiply by 10?

*In the transcripts below, underlined words indicate vocal stress and nonverbal or interpretative information appears in parentheses.
Larry: Because every one centimeter there’s 10 millimeters.

Teacher: Every one centimeter, there’s 10 millimeters. Anybody have a question, stay put Larry. Anybody have a question about that?

Student: Yeah...

Teacher: Did everybody get that? Did everybody get 170 millimeters squared for their answer?

Student: No.

Teacher: How many of you got 170? How many of you got something else? If you got something else I don’t understand why your hand isn’t in the air asking him how he got it. Larry, call on people.

Larry: ’cause there’s 17.5 and the point 5 would make it real complicated so I just add that and for every centimeter, then there’s one millimeter.

Eliza: Uh, uh. (No.)

Larry: I mean for every centimeter, there’s 10 millimeters.

This passage illustrates the collaborative construction of one of the arguments supporting the first mathematical position. Although Larry is ultimately accountable for the mathematical content of the argument, the teacher provides assistance by questioning him in a way that allows Larry to unpack the different components of his argument.

From a rhetorical perspective, the first thing that Larry does in response to the teacher’s query is to set forth the grounds for his solution to the area problem by providing a measure in squared centimeters (which corresponds to the number of grid squares enclosed by the geometrical shape). Then, the teacher revoices Larry’s grounds via repetition and expansion — adding the relevant unit of measure (“17 1/2 squared centimeters, you mean”). Far from being merely stylistic, this revoicing highlights a critical feature of the task. The revoicing also forces Larry to agree or disagree, thereby aligning him with a position that takes the unit of analysis seriously. Larry signals his agreement and proceeds, after an acknowledgement by the teacher (“Okay”), to introduce the warrant of his argument. This warrant takes the form of a multiplication algorithm (17 × 10) that allows him to find the unknown area in millimeters squared from the area measured in centimeters squared. By prompting for a backing of such an algorithm (“Why did you multiply by 10?”), the teacher is allowing Larry to self-correct the discrepancy between the previous ratio (1:1) and the algorithm (17 × 10 not 17 × 1). Larry responds by introducing a different ratio (1:10) thereby eliminating one of the inconsistencies in his explanation but also providing the necessary support for his claim (170). (The structure of Larry’s argument is illustrated in Fig. 1.)

Interestingly, it is not until all the components of Larry’s argument are laid out that
the teacher proceeds to revoice both Larry’s backing (“Every one centimeter, there is 10 millimeters”) and claim (“How many of you got 170?”). In this way, she signals that one position has been articulated, that Larry is aligned with that position, and that other positions may now be identified. By not correcting the inconsistencies in Larry’s position, she also indicates that her role will be as discussion orchestrator more than as explanation evaluator.

In addition, she repeatedly challenges the rest of the class to cross-examine Larry about his position (“Anybody have a question, stay put Larry. Anybody have a question about that?”). These repeated invitations serve the purpose of encouraging others to question the veridicality of his position. They also put Larry in the role of defender of the first position, not merely the articulator of that position. Although Larry was assisted by the teacher throughout the development of this first position, now Larry is held accountable for its mathematical content.

In the following turn (23), Larry provides a side argument to explain away the inconsistency between his claim (170 mm²) and the grounds initially introduced (17.5 cm²). He explains that he rounded 17.5 to 17 to make the computation easier and reasserts that the ratio between centimeters and millimeters is 1:1. Thus, he self-corrects one inconsistency (between his claim and grounds) and reintroduces the other inconsistency (between one backing and another; one ratio and another) in his position. These repeated inconsistencies make his position difficult to follow but the self-corrections indicate that he is aware of the importance of clarifying his statements to achieve greater coherence and/or social acceptance. Eliza points out her disagreement with his side argument (or perhaps his entire argument) and Larry responds immediately by correcting the backing ratio to 1:10. Thus, Eliza shows her acceptance of the role of evaluator — a role that is more typically assumed by the teacher.

Figure 1. Larry’s argument.
Eliza’s Argument

Larry’s statement that there is 10 millimeters for every centimeter constitutes a pretext for Eliza to introduce a new mathematical argument based on the 1:100 ratio.

26 Eliza: It would be if every centimeter is 10 of those then, the dimensions of a millimeter would be 10 by 10, right? So that makes every centimeter worth 100 millimeters, right?

27 Teacher: A hundred square millimeters.

28 Eliza: Yeah, a hundred square millimeters, so if you add them all, each centimeter being a hundred, you would get 17 hundred.

Rhetorically speaking, Eliza concentrates on challenging Larry’s backing, i.e. the 1:10 ratio for area units, and adjusting correspondingly the initial warrant and claim. However, Eliza not only states a new ratio (1:100) as backing for her position, but also provides an additional sub-argument to support it as if she understands that the challenger of a previous position should carry the burden of proof. In this sub-argument, Eliza introduces the contrast between area and linear measurement. She proposes that the area ratio between square centimeters and square millimeters is 1:100 given that the linear ratio is 1:10 (backing), and that one needs to multiply the dimensions (in millimeters) of a square centimeter to find its area in square millimeters (warrant): “It would be if every centimeter is 10 of those then the dimensions of a millimeter would be 10 by 10, right? So that makes every centimeter worth 100 millimeters, right?” Once she establishes the plausibility of the new backing, she continues by adjusting both the warrant and the claim: “So if you add them up, each centimeter being a hundred (warrant), you would get 17 hundred (claim).”

Although Eliza challenges Larry’s position by substituting a different backing that changes both the warrant and the resulting claim, she does not question Larry’s grounds of 17 (and not 17 1/2). In so doing, she agrees to frame the argument in accordance with Larry’s side argument about rounding. It is worth noticing that, unlike Larry, Eliza receives little assistance from the teacher, who merely revoices via expansion her backing to include the correct unit of measure “square millimeters”. Like Larry, Eliza agrees with the teacher’s revoicing and reinforces her agreement (which is a crucial piece of her position) by repeating the unit of measurement. As we saw with Larry, this revoicing move requires Eliza to agree or disagree. Her subsequent agreement with the area unit of measurement is both consistent with her position and clarifies it. In contrast, Larry’s earlier agreement with the area unit of measurement is inconsistent with his position which appears to confuse linear and area measurement. (A representation of Eliza’s argument is given in Fig. 2.)

Todd’s Contribution to Larry’s Argument

Immediatley after Eliza presents her position, Todd addresses the issue of rounding.

29 Todd: Do you know when you come to like the end of something and
Unlike Eliza, Todd agrees with Larry on both the ratio (backing) and the conversion procedure (warrant), but disagrees on the grounds. His intervention is then aimed at persuading Larry that dropping the .5 from the measure of the number of grid squares is unnecessary since, in order to multiply it by 10, one can simply move the decimal over one place. (Todd’s intervention presupposes agreement that a fraction such as $17\frac{1}{2}$ can be re-expressed as a decimal such as 17.5.) Instead of explicitly indicating the direction of movement of the decimal (e.g. right), Todd gestures by swinging his arms in the desired direction. The confusion between the two (”Which way?”) is due to the fact that they are facing each other and Todd’s right is Larry’s left. As a result of this brief exchange, Larry modifies his initial claim from 170 to 175. Unlike Eliza, who focuses on the ratio (the backing), Todd is exclusively concerned with the numerical accuracy of the original area computation (the grounds). His concern suggests that he fails to frame the instructional activity as one in which one should question whether the 1:10 ratio for linear measurement
applies as well to area measurement. This failure to frame the problem in the same way that Eliza and Larry do will surface later in the discussion and be explicitly addressed at that point. After the exchange between Todd and Larry, Eliza reaffirms her disagreement with Larry’s claim by revoicing it and then denying its validity “175? Nope.”

The Teacher’s Summary of the Collective Argument

After the exchange between Todd and Larry and Eliza’s disagreement with their position, the teacher summarizes the two arguments and invites her students, and in particular Nancy, to contribute to the debate.

37 Teacher: We still have some more discussion here. Eliza insists that it’s, each one is not 10 but it’s a 100. Todd is agreeing that it’s 10. Nancy?

With this conversational turn, the teacher clearly articulates her view of the crucial differences in the two arguments: their contradictory backings. This is consistent with her stated goal of calling attention to student generalizations. She also aligns students with argumentation positions: Larry and Todd with the one position, Eliza with the other. She does this via reported speech (“Eliza insists”, “Todd is agreeing”). As O’Connor and Michaels argue, reported speech is a powerful tool for depicting the students’ talk in ways that may serve the teacher’s agenda. By using the verbs “insists” or “agrees”, she “animates the student as the originator of the intellectual content of the revoiced utterance, even though the teacher may have reformulated it” (O’Connor & Michaels, 1996, pp. 79–80). Notice that Ms. Kingsley aligns Todd with Larry’s backing and that she ignores Todd’s insistence upon the inaccuracy of Larry’s grounds. In this way, she indicates her own alignment with the way Larry and Eliza have framed the argument and not with Todd’s framing of it. This is one of the ways a shared frame for the classroom community is created. Through the contextualization cue of revoicing, the teacher identifies the frame as consisting of two opposing factions of students, and two opposing backings, warrants, and claims.

Nancy’s Expansion of Eliza’s Argument

When Nancy is prompted by the teacher to participate, she provides an argument that can be considered an expansion of Eliza’s argument in support of her backing, i.e. the 1:100 ratio between area units. “The things are 1 by 1 and 10 by 10, it’s a hundred each,” Nancy says, modifying slightly the warrant of Eliza’s sub-argument and stating in very explicit terms the conversion procedure derived from the 1:100 ratio. She also contrasts her algorithm again explicitly, with that used by Larry and Todd: “So you multiply by 100 instead of 10.” (A representation of Nancy’s main argument is given in Fig. 3.)

Next, responding to a request by the teacher to show the class which she means, Nancy rephrases her sub-argument in support of the 1:100 area ratio using a visual representation on the overhead projector. Before Nancy begins her explanation, Ms. Kingsley tells her
to make sure that all the students are ready to listen, “Wait ‘till you have their attention… because this is important for people to understand.” After waiting for the required attention, she puts a $10 \times 10$ centimeter grid square on the overhead and asks the class to imagine that the entire $10 \times 10$ grid square is a single square centimeter (divided into square millimeters) “blown up”. In addition, she sweeps her pencil across the grid first horizontally then vertically to indicate how 10 millimeters by 10 millimeters results in a ratio of 1:100.

48 Nancy: This is like one of the little squares on here. Okay? This is 1 centimeter by 1 centimeter. (Shows a single centimeter square on the overhead projector.) Okay, like if you blow this up, okay, to get your millimeters, this is your millimeters right here (shows a grid containing 100 centimeter squares but suggests it should be viewed as a single centimeter square divided into millimeter squares) and this is 1 centimeter by 1 centimeter. This has to be 10 millimeters by 10 millimeters so that way you have a hundred millimeters in one of these, one of these little things right here. There’s a, this (the grid of 100 squares) is just this (the single centimeter square) blown up.

49 Teacher: How many times is it blown up? How many times bigger is that?

50 Nancy: It’s a hundred times bigger.

51 Teacher: It’s a hundred times bigger in area.

52 Nancy: But, so this is, that means that each one of the, each one of these little. That means each one of these boxes, like right here (shows the original area problem and points to one of the grid squares), is
just like one of these (shows the grid of 100 squares), just like smaller. That means each of these boxes (shows the original problem) is worth a hundred millimeters

53 Teacher: squared

54 Nancy: so that means if there’s 17.5 centimeters, there’s 1,750 millimeters.

55 Eliza: (claps) Bravo, bravo, I agree with you 100%.

By using this visual representation, Nancy sets forth a backing for her warrant which allows one to derive the area ratio from the linear ratio: a squaring operation is required because two linear dimensions are involved in area. This backing was part of Eliza’s argument as well (turn 26) but without the visual support that Nancy provides. Once the 1:100 ratio is thoroughly supported, both numerically and visually, Nancy returns to her main argument and concludes: “So that means if there’s 17.5 centimeters (grounds), there’s 1,750 millimeters (claim).”

After Nancy explains the ratio both visually and verbally, Ms. Kingsley asks her to repeat the ratio, “How many times is it blown up?” and to state what the result would be, “How many times bigger is that?” Nancy repeats her result, “It’s a hundred times bigger” and the teacher revoices through expansion this explanation to “It’s a hundred times bigger in area.” The teacher also expands Nancy’s statement before her concluding claim to include an area unit of measurement “squared”. Thus, through these discussion orchestration moves, the teacher highlights the critical features of the mathematical content frame (e.g. emphasis on the differences between linear and area measurement) as well as critical features of the instructional process frame (e.g. sharing of instructional responsibilities).

Nancy’s conclusion is dramatically approved by Eliza with clapping hands and expressions of “bravo”. In addition, the teacher signals her agreement by laughing and not sanctioning Eliza’s theatrical display. Thus, a similar attitudinal frame toward Nancy’s performance is conveyed by both Eliza and Ms. Kingsley. At this point in the lesson, the classroom discussion not only takes on the tone of a scientific argument but also takes on the cast of a climactic theatrical event. After the applause, Ms. Kingsley asks whether everyone is as convinced of the validity of Nancy’s argument as she and Eliza are.

56 Teacher: How many of you agree with her and see that? Because if you don’t, we need to go further. How many of you still do not agree with that? How many of you still think it’s 175? It’s okay to think differently. Okay, we still need some more.

Todd’s Misunderstanding of the Framing of the Problem

In response to the teacher’s query, Todd brings up a previous lesson on converting units in linear measurement. He expresses his belief (based on that previous lesson) that to
convert centimeters to millimeters one has to move the decimal over (i.e. to the right) one place (warrant) because the ratio is 1:10 (backing no. 1) and they were “doing powers of ten” (backing no. 2). (The structure of Todd’s argument is shown in Fig. 4.)

57 Todd: I thought it took 10 millimeters to make up a centimeter.

58 Teacher: Okay, does it take 10, where did you decide it took 10 millimeters?

59 Todd: Because you move the decimal over.

60 Teacher: Oh, so you moved the decimal over. You moved the decimal.

61 Todd: Yeah, because that’s what we were doing powers of ten.

62 Teacher: And what kind of measurement were we doing then? Were we doing area or length?

63 Students: Length.

64 Teacher: Okay, and, and is the length 10 times bigger?

65 Todd: Yes.

66 Teacher: Okay, so you agree with that? Our discussion is whether or not the area is just 10 times bigger. Does the same thing happen with the area? We didn’t do area. Okay? Yes?

The teacher’s response to Todd’s belief in the 1:10 ratio is critical for bringing the

Figure 4. Todd’s argument.
debate to a successful closure. Instead of providing Todd with the correct ratio for the area units, the teacher decides to question Todd in a way that requires him to deploy crucial components of his argument. The teacher does not rule out completely the validity of Todd’s warrant (“move decimal over one place”) and backings (“powers of ten” and “1:10 ratio”) for that previous lesson, but rather chooses to call into question their relevance for this lesson (“And what kind of measurement were we doing then?”; “Does the same thing happen with the area?”). In this way, she invites other students to clarify the frame for the current lesson (area measurement vs. linear measurement). Thus, she tries to get Todd to understand that the way the lesson is framed by the classroom community influences the relevant warrants and backings of the argument.

This exchange between Todd, the teacher, and the rest of the class ends with another student requesting permission to explain to Todd privately the distinction between area and linear measurement. In this way, this other student indicates her alignment with the teacher’s frame for this lesson (both in terms of the content and in terms of sharing instructional responsibilities). The exchange also illustrates Schiffrin’s proposal that misunderstandings can signal a lack of a shared definition of the situation.

Further Elaboration by Nancy of Eliza’s Argument

At this point, Nancy volunteers to provide another explanation. She does not limit herself to a literal restatement of her original position, but builds a bridge between her explanation and Todd’s.

71 Nancy: Cause like he was saying that each one of these is 1 centimeter by 1 centimeter (shows both the original problem and the single centimeter square) so this is 10 millimeters by 10 millimeters. That’s all there is because each of these is 1 centimeter, that way, the length is, is, you move the decimal place one space for each length and there’s two lengths.

72 Teacher: Oh, so you’re saying that you’re moving it two powers of ten when you’ve got 1,750?

73 Nancy: Yeah, because you have two sides that you multiply times 10.

74 Teacher: 10 times 10...

Nancy makes this connection between her explanation and Todd’s by using reported speech (“‘cause like he was saying”). The teacher reinforces the connection by reporting Nancy’s speech while revoicing Todd’s terminology (“so you’re saying that you’re moving it two powers of ten”). The teacher also revoices Nancy’s argument through expansion (“10 times 10††”). (The structure of Nancy’s sub-argument is presented in Fig. 5.)

In summary, Nancy’s argument is unique in various ways. Her argument is not only more explicit than Eliza’s, but also more complex in terms of the number of sub-arguments and the number of warrants and backings. Nancy supports her speech with a visual rep-
presentation which illustrates her warrants and backings. In addition, she incorporates into her own argument a modified version of Todd’s warrant (i.e. move the decimal over one place). Her position was facilitated in a number of ways by the teacher. Before Nancy illustrated her position using a visual display, the teacher told her to wait until the attention of the entire class was focused on her. In addition, Ms. Kingsley revoiced through expansion and reported speech Nancy’s claim and warrant. The teacher also promoted argument integration by rephrasing Nancy’s ideas using terms previously introduced by Todd. Thus, the collective argument provided by Nancy and her teacher contained more explicit references to the arguments of other speakers — by adopting their terminology or by expanding and illustrating information left implicit or by addressing the flaws present in the arguments of other participants. This resulted in an argument that was not only longer, more complete, and correct, but also more highly integrated with previous arguments. In addition, Ms. Kingsley increased the salience of Nancy’s argument by recruiting the attention of the rest of the class and by repeating and revoicing her speech.

Discussion

What can we say about discussion orchestration and argumentation in classrooms based upon our analysis of a single classroom lesson by one middle school mathematics teacher? We believe that this analysis has allowed us to depict, in detail, how middle school students from a low-income, urban neighborhood conducted a collective argument, an accomplishment that theorists and educators believe is important to scientific practice. As Lampert proposed, the process of argumentation in this lesson involved the examination of premises (“Were we doing area or length?”), disagreement (170 vs. 175) and counter-arguments (1:10 vs. 1:100).

As a participant in a reform mathematics project, Ms. Kingsley expressed her intentions to help students take an active role in classroom discussion. She also wanted them to explore the generality of mathematical concepts more than to use algorithms accurately.
We will refer to the first goal as the teacher’s instructional process frame and the second goal as her mathematical content frame. What does our analysis indicate about her ability to implement these goals in her classroom? We will deal with the instructional process goal first.

The analysis showed how Ms. Kingsley was able to accomplish her instructional goals of being nondirective in her teaching and getting students actively involved in explaining their ideas, listening to each other, and evaluating their own and each other’s arguments. Instruction in this classroom during the target lesson was quite different from more typical school mathematics practice. In school mathematics, lessons often follow a teacher-dominated instructional script where the teacher initiates with a question or explanation, the students respond with an answer or question, and the teacher evaluates (or I–R–E, according to Mehan, 1979). This is especially true for students from economically disadvantaged backgrounds (Dossey et al., 1988; Silver et al., 1995). We saw that, in this lesson, students were more likely than the teacher to initiate explanations, to provide answers or claims backed up by their explanatory grounds, warrants, and backings, and to evaluate their own and each other’s arguments. In addition, the teacher participated in the collective argument by recruiting attention and participation from the class, aligning students with positions through reported speech, highlighting positions through repetition, and pointing out implicit but important aspects of the explanation (e.g. units of measurement) through expansion. There appeared to be a working consensus in the classroom about this alternative way to frame the instructional processes since no obvious misunderstandings about social roles occurred.

The mathematical content frame, in contrast, showed more disagreement among the students than did the instructional process frame. Toulmin’s scheme for displaying the organization of arguments helped us see the contrasts between the different framings of the argument in terms of facts or grounds (17 vs. 17.5 grid squares), algorithms or warrants (multiply by 10 or 100), premises or backings (1:10 vs. 1:100) as well as solutions or claims (170 vs. 1700 vs. 1750 vs. 175). We saw that Todd and Larry framed the overall task goal differently from Nancy and Eliza: as a lesson about linear measurement not area measurement. Todd also framed a task subgoal differently from Larry because Larry was content to round his answer up to the closest integer while Todd thought it was important to convert fractions to decimals before multiplying. Eliza and Larry seemed to agree that rounding one’s answer was an acceptable procedure while Todd and Nancy agreed to use decimals. Thus, in this instance, the mathematical content frame that includes overall task goals and subgoals kept changing in response to subtle shifts in social norms and expectations.

Ms. Kingsley did not explicitly evaluate either the rounding or the decimal procedures but she did intervene in framing (both indirectly through other students and directly) the overall task goal: the appropriate measurement topic of this lesson. Todd expressed his confusion with the ratio that Nancy presented in her argument because it contradicted his memory of a previous lesson. The teacher asked the class to help Todd remember the topic of that lesson: “What kind of measurement were we doing then? Were we doing area or length?” She also explicitly identified the current topic, “Our discussion is whether or not the area is just 10 times bigger. Does the same thing happen with area? We didn’t do area.” This intervention was consistent with another one of her goals: to help students
 generalize their knowledge appropriately. She wanted her students to question whether the ratio that works for linear measurement also works for area measurement.

In conclusion, we believe that teachers need more detailed examples of successful alternatives to the I–R–E instructional process frame in order to understand how to change their practice to conform to the recommendations of educational reformers in North America and Europe. Examples can also provide information about how to evaluate the adequacy of mathematical arguments. Examples drawn from schools that serve low-income communities are especially valuable in demonstrating the successful implementation of alternative instructional processes. As changes in classroom instructional practices occur, it is important that the implementation and success of these reforms be evaluated. We believe that classroom discourse analysis has the potential for becoming a powerful tool for assessing the impact of educational reform in mathematics.

Recent advances in educational theory have shifted the focus of investigation from the individual thinker to the person who participates as a member of a community. This view is remarkably consistent with writings in the philosophy of science which argue for the importance of the social norms and practices of the scientific community in the validation of scientific theories. That is, the acceptance of explanations in scientific domains is based not only on logical or empirical criteria but on whether an argument is capable of persuading other members of the community. Since this view of argumentation is intimately bound to context, mathematics can no longer be seen as a purely individual epistemic activity but rather as an activity which is social in a principled way. Educational research on collective argumentation in classrooms can help us understand how to assist students in their appropriation of this crucial scientific practice.

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